A Precarious Existence: Population Growth of Sperm Whales

Project Module Associated with
Introduction to Computational Science:
***Modeling and Simulation, 2nd Edition* by**
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Prerequisites for Project 1: *Module 13.3, "Time after Time: Age- and Stage-Structured Models"*

Problem Introduction

Although found in salt waters throughout the world, sperm whales (*Physeter microcephalus*), have had a precarious existence in the past several hundred years. From the 1600's until 1981, whalers have hunted the animal. Moreover, sperm whales are susceptible to other threats, such as shipping, ingestion of debris, entanglement in fishing gear, noise, and chemical pollution, including oil spills. In this module, we employ stage-based models to discover the asymptotic growth rate under certain circumstances and the sensitivity of that rate to various parameters.

Chiquet et al. enumerated five stages for female sperm whales (2013):

- Stage 1, calf, approximately the first two years
- Stage 2, juvenile/immature, approximately seven years
- Stage 3, mature female, who transitions to Stage 4 after giving birth. A mature female remains in Stage 3 if she is not able to reproduce.
- Stage 4, mother, who is caring for a calf for about two years before transitioning to Stage 5
- Stage 5, post-breeding female, who is not a mother and does not breed. After this interval, usually lasting for several years, the female returns to Stage 3.

The researches considered three basic parameters: the fecundity rate, or average number of female pregnancies per year by a mature female (b); the probability of surviving in Stage i (σ_i); and the probability of transitioning from one stage to another (γ_i). For i = 1, 2, 3, and 4, γ_i is the probability that the female sperm whale transitions from Stage i to Stage (i + 1), while γ_5 is the probability that she moves from Stage 5 (post-breeding) to Stage 3 (mature). To remain in Stage i, the whale must survive and not move to another stage. Thus, the probability, P_i, of a whale remaining in Stage i is the product of the probability of surviving in that stage and the probability of *not* moving to another stage. Also, the fraction, G_i, of Stage i female whales that transition to the next stage ($1 \rightarrow 2$ for G_1, $2 \rightarrow 3$ for G_2, $3 \rightarrow 4$ for G_3, $4 \rightarrow 5$ for G_4, and $5 \rightarrow 3$ for G_5) is the probability that such a whale survives and, if alive, moves to the next stage. From the literature and calculations, Chiquet et al. (2013) estimated the parameters in Table 1 with a basic unit of time of one year.

Parameter	**Description**	**Estimated Value**
b	fecundity rate	0.1250
σ_1	probability of surviving Stage 1	0.9070
σ_2	probability of surviving Stage 2	0.9424
σ_3	probability of surviving Stage 3	0.9777
σ_4	probability of surviving Stage 4	0.9777
σ_5	probability of surviving Stage 5	0.9777
γ_1	probability of moving from Stage 1 to Stage 2	0.4732
γ_2	probability of moving from Stage 2 to Stage 3	0.1151
γ_3	probability of moving from Stage 3 to Stage 4	0.2586
γ_4	probability of moving from Stage 4 to Stage 5	0.4920
γ_5	probability of moving from Stage 5 to Stage 3	0.4920

Table 1 Estimated parameters with a basic unit of time of one year (Chiquet et al. 2013)

Projects

1.
- **a.** Draw a state diagram for female sperm whales.
- **b.** Derive a Lefkovitch matrix from the problem description above.
- **c.** Assuming an initial female sperm whale population distribution of (70, 171, 297, 147, 138) for Stages 1-5, determine the population distribution after 20 years.
- **d.** Estimate the vector of percentages to which the category distributions converge.
- **e.** Determine the projected annual growth rate, λ.

2.
- **a.** Derive a Lefkovitch matrix from the problem description above.
- **b.** Determine the projected annual growth rate, λ.
- **c.** Perform a sensitivity analysis to determine the parameters σ_i to which the growth rate λ is most sensitive.
- **d.** Perform a sensitivity analysis to determine the parameters γ_i to which the growth rate λ is most sensitive.
- **e.** Discuss the results from Parts c and d and make conservation recommendations.

3. **a.** Derive a Lefkovitch matrix from the problem description above.
b. Determine the projected annual growth rate, λ.
c. Perform a sensitivity analysis to determine the parameters P_i to which the growth rate λ is most sensitive.
d. Perform a sensitivity analysis to determine the parameters G_i to which the growth rate λ is most sensitive.
e. Discuss the results from Parts c and d and make conservation recommendations.

4. Environmentalists are concerned with the sublethal effects on sperm whales of catastrophic events, such as the British Petroleum (BP)'s oil rig explosion and oil spill in the Gulf of Mexico in 2010. In this project, we consider the impact on the population distribution of a reduction in the birth rate (Chiquet 2017).
a. Derive a Lefkovitch matrix from the problem description above.
b. Assuming an initial female sperm whale population distribution of (70, 171, 297, 147, 138) for Stages 1-5, determine the population distribution and total population after 50 years.
c. Suppose that the fecundity rate (b) of such a disaster is reduced by r. Determine the value of r that results in a projected annual growth rate, λ, less than one. Thus, what is the critical reduction percentage? What is the projected population when $\lambda < 0$? Discuss the results.
d. Assuming an initial female sperm whale population distribution of (70, 171, 297, 147, 138) for Stages 1-5 and the parameters in Table 1, determine the population distribution after 20 years. Then, starting with this distribution and a value for b that yields $\lambda < 0$ (see Part c), run the simulation for another 5 years. Finally, using the resulting population distribution and the parameter values from Table 1, run the simulation for an additional 175 years. Compare this distribution and population total with those from Part b and discuss the results.

5. Environmentalists are concerned with the lethal effects on sperm whales of catastrophic events, such as the British Petroleum (BP)'s oil rig explosion and oil spill in the Gulf of Mexico in 2010. In this project, we consider the impact on the population distribution of reductions in the probabilities of surviving (Chiquet 2017).
a. Derive a Lefkovitch matrix from the problem description above.
b. Assuming an initial female sperm whale population distribution of (70, 171, 297, 147, 138) for Stages 1-5, determine the population distribution and total population after 200 years.
c. Suppose from such a disaster the probability (σ_i) of surviving Stage i is reduced by $2r$ for the first two more vulnerable stages and by r for Stages 3-5. Determine the value of r that results in a projected annual growth rate, λ, less than one. Thus, what is the critical reduction percentage? What is the projected population when $\lambda < 0$? Discuss the results.

d. Assuming an initial female sperm whale population distribution of (70, 171, 297, 147, 138) for Stages 1-5 and the parameters in Table 1, determine the population distribution after 20 years. Then, starting with this distribution and a set of σ_i values that yields $\lambda < 0$ (see Part c), run the simulation for another 5 years. Finally, using the resulting population distribution and the parameter values from Table 1, run the simulation for an additional 175 years. Compare this distribution and population total with those from Part b and discuss the results.

References

Chiquet, Ross A. 2017. "Analysis of Lethal and Sublethal Impacts of Environmental Disasters on Sperm Whales Using Stochastic Modeling." 2017 Joint Mathematics Meeting.

Chiquet, Ross A., Baoling Mab, Azmy S. Ackleha, Nabendu Pala, and Natalia Sidorovskaiac. 2013. "Demographic analysis of sperm whales using matrix population models." *Ecological Modelling*, Volume 248, 10 January 2013, 71–79. http://dx.doi.org/10.1016/j.ecolmodel.2012.09.023.