Under Attack from All Sides: Modeling Populations Having Infected Prey

Project Module Associated with
2nd Edition, Introduction to Computational Science:
Modeling and Simulation **by**

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Project 1 Prerequisite: *Module 2.3, "Constrained Growth and Decay;" Module 4.2, "Predator-Prey Models;" and Module 4.3, "Modeling the Spread of SARS—Containing Emerging Disease"*

Description of Problem and Assumptions

Haque et al. considered the impacts on the number of predators and prey, where an infectious disease is presence in a prey population (2009). In the project of this module, we model such a system, making several assumptions that match reality for many populations and diseases, such as infected prey do not recover, reproduce, or compete for resources. Moreover, although being more likely to catch vulnerable, infected prey, the predators do not derive as much nutrition from these weakened animals. With a designated carrying capacity, we employ a constrained-growth model for the prey population. We only consider spread of disease in the prey population. Considering a large prey population with few infected individuals, a sound model for the rate of change of individuals flowing from susceptibles to infecteds is a fraction of the entire prey population times the number of possible interactions of susceptibles and infecteds.

In the project, we will see how various parameter selections yield different results, such as extinction of all participants, elimination of diseased individuals, or oscillations. Table 1 gives ranges of parameters with the basic unit of time being one week, where S is the number of susceptible prey, I is the number of infected prey, and P is the number of predators (Haque et al. 2009).

Parameter	**Symbol**	**Range of Values**
Death rate of infecteds due to disease	μ	$0.1 \le \mu \le 0.8$
Infected prey death constant due to predation	m	$0.1 \le m \le 0.5$
Infection constant	λ	$0 \le \lambda$
Initial number of infected prey	I_0	0.7
Initial number of predators	P_0	1.2
Initial number of susceptible prey	S_0	1.2
Predator birth constant due to eating infected prey	θ_2	$0 \le \theta_2 \le 0.1$
Predator birth constant due to eating susceptible prey	θ_1	1
Predator death rate	δ	0.4
Prey carrying capacity	K	$25 \le K \le 35$
Susceptible prey death constant due to predation	c	$1 \le c \le 5$
Susceptible prey growth constant	r	9

Table 1 Model's parameters with basic unit of time being one week (Haque et al. 2009)

Exercises

1. **a.** On paper, develop a system of differential equations to model the rates of change of predator (P), susceptible prey (S), and infected prey (I) populations, using the assumptions in the section "Description of Problem and Assumptions."

b. Show your work to find two equilibria for the system, where $I = P = 0$. Express each equilibrium as a triple of values, (S, I, P).

c. Show your work to find a system equilibrium with $I = 0$, where S and P are non-zero.

d. Show your work to find a system equilibrium with $P = 0$, where S and I are non-zero.

e. This part involves finding a system equilibrium, where S, I, and P are non-zero. First, setting dP/dt equal to zero, solve for I in terms of S. Then, setting dI/dt equal to zero, solve for P in terms of S and I. Consequently, using the result of our first step, we could solve for P in terms of S. Finally, set dS/Dt equal to zero. By substituting for P and I from the first two steps, we can obtain a quadratic equation only in terms of S, which we can solve using the quadratic formula. Show your work on all the steps, but do not complete the process of solving the quadratic equation.

Project

1. **a.** Develop a system dynamics model of predator (P), susceptible prey (S), and infected prey (I) populations, using the assumptions in the section "Description of Problem and Assumptions."

b. Graph P, S, and I versus time for 30 weeks using the following parameter values: $K = 25$, $c = 3$, $m = 0.3$, $\mu = 0.5$, $\theta_1 = 1$, $\theta_2 = 0.1$, $\delta = 0.4$, $\lambda = \mu + r +$

0.05 = 9.55. Describe and discuss the results. Give an equilibrium to which the graphs converge (see Exercise 1).

c. Graph P, S, and I versus time for 150 weeks using the parameter values of Part b except $\lambda = 0.3$ and $\theta_2 = 0.05$. Describe and discuss the results. Substitute the parameters into the equilibrium from Exercise 1c. Does the triple of values, (S, I, P), converge to this equilibrium?

d. Graph P, S, and I versus time for 150 weeks using the parameter values of Part c except $\lambda = 1.54$. Describe and discuss the results. Substitute the parameters into the equilibrium from Exercise 1c. How do the number of susceptibles and the number of predators behave around the S and P values of this equilibrium?

e. Graph P, S, and I versus time for 150 weeks using the parameter values of Part a except $\lambda = 0.3$, $m = 1$, and $P_0 = 2$. Describe and discuss the results. Substitute the parameters into the equilibrium from Exercise 1c. How do the number of susceptibles and the number of predators behave around the S and P values of this equilibrium?

References

Farrell, A. P., James P. Collins, H. R. Thieme. 2017. “Prey-predator-parasite: an ecosystem model with fragile persistence,” presentation at Joint Mathematics Conference.

Haque, M., J. Zhen, E. Venturino. 2009. “An ecoepidemiological predator-prey model with standard disease incidence,” *Math. Meth. Appl. Sci.*, 32 (2009), 875–898.