Modeling the Formation of Fur Patterns

Project Module Associated with
***2nd Edition, Introduction to Computational Science: Modeling and Simulation* by**
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Prerequisite: One of Module 10.3 on "Spreading of Fire," Module 10.4 on "Movement of Ants–Taking the Right Steps," or Module 10.5 on "Biofilms: United They Stand, Divided They Colonize"

For one cellular automaton simulation of the formation of the patterns in fur, each cell has two states, differentiated (D), or colorful, and undifferentiated (U), or not colorful. We can visualize a cell in a colorful state as white and a cell in a non-colorful state as black. Initially, we assign the states of cells at random with *initialDensity* being the density of D cells.

At each time step, we update the color of every cell. Suppose a cell *C* is in row and column *h* and *k*, respectively. An activator for *C* is a differentiated (white) cell in an inner ellipse around *C*. An inhibitor for *C* is an undifferentiated cell outside this inner ellipse but inside an outer ellipse. Let *A* and *I* be the number of activators and inhibitors, respectively, for *C*; and let *ratio* be a parameter between 0.0 and 1.0 that indicates the power of the inhibitors. If for cell *C* we have *A – ratio * I* being positive, then at the next time step *C* becomes white (D). However, if the expression *A – ratio * I* is negative, *C* becomes black (U).

We will consider each inner and outer ellipse to be parallel to the horizontal (*x*) or vertical (*y*) axis. Suppose *a* is the distance from the center of an ellipse to a vertex in the *x* direction, while *b* is the distance from the center to a vertex in the *y* direction. The equation of an ellipse with center (*h*, *k*) is as follows:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

A point (*x*, *y*) is inside the ellipse if the expression on the left is less than zero; and if the expression on the left is greater than 0, the point is outside the ellipse.

A model could have the following parameters:

- *n* – size of square hide
- *innerRadiusX* between 1 and 10, inclusive
- *innerRadiusY* between 1 and 10, inclusive
- *outerRadiusX* between 1 and 10, inclusive
- *outerRadiusY* between 1 and 10, inclusive

- *initialDensity* between 0.0 and 1.0, inclusive
- *ratio* between 0.0 and 2.0, inclusive
- *t* – number of time steps

Projects

1. **a.** Develop a cellular automaton simulation of the formation of the patterns in fur as described in the module.

Run the simulation with n = 61, less than 10 times steps, and the following parameter values. Describe the results:

b. innerRadiusX = innerRadiusY = 3, outerRadiusX = outerRadiusY = 6, initialDensity = 0.50, and ratio = 0.35

c. innerRadiusX = innerRadiusY = 3, outerRadiusX = outerRadiusY = 6, initialDensity = 0.50, and ratio = 0.47

d. innerRadiusX =2, innerRadiusY = 5, outerRadiusX = outerRadiusY = 7, initialDensity = 0.50, and ratio = 0.26

e. innerRadiusX =5, innerRadiusY = 2, outerRadiusX = outerRadiusY = 7, initialDensity = 0.50, and ratio = 0.26

f. Explore the results of other parameter sets.

p

References

Wilensky, U. (2003). NetLogo Fur model. http://ccl.northwestern.edu/netlogo/models/Fur. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.

Wilensky, U. (1999). NetLogo. http://ccl.northwestern.edu/netlogo/. Center for Connected Learning and Computepr-Based Modeling, Northwestern University, Evanston, IL.