# Osos Pardos de Espana - Brown Bears of Spain 

Project Module Associated with<br>$2^{\text {nd }}$ Edition, Introduction to Computational Science:<br>Modeling and Simulation by

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Projects 1-5, Age-Structured Modeling Prerequisite: Module 13.3, "Time after Time: Age- and Stage-Structured Models"
Projects 6-11, Agent-Based Modeling Prerequisite: Module 11.2, "Agents of Interaction: Steering a Dangerous Course"
Projects 6-11, Cellular Automaton Modeling Prerequisite: One of Module 10.3 on "Spreading of Fire," Module 10.4 on "Movement of Ants-Taking the Right Steps," or Module 10.5 on "Biofilms: United They Stand, Divided They Colonize"

## Introduction

The brown bear (Ursus arctos) is distributed widely, found in Europe, Asia, and North America. In the North America, this species is sometimes known as the grizzly bear. A brown bear is large, with a massive shoulder hump on its back formed by a concentration of muscles that enable the animal's remarkable ability to dig.

Where bear and human populations overlap, there is often conflict. Although we may think of bears as major predators, the brown bear is an omnivorous animal, utilizing plants as a major source of nutrition. Bears are intelligent creatures, and many have adapted to the human activity, feeding on human food sources.

In northern Spain, the Cantabrian Mountains stretch to the west of the Pyrenees range for about 180 miles, parallel to the coast. Along the slopes of these mountains are small populations of brown bears. With loss of habitat and hunting pressures during the twentieth century, their populations declined rather precipitously. In 1997, Wiegand, et al estimated that there were only $50-60$ individuals. However, conservation efforts led by the Fundación Oso Pardo (Brown Bear Foundation) have met with success, and the population increased to 200 by the 2013 estimates.

Recovery for bear populations is challenging. Brown bears may become sexually mature after 4.5-7 years, but because younger males must compete with older, stronger males for mates, they may be 8-10 years old for reproductive success. Mating takes place during late spring to mid-summer. However, the fertilized egg is not implanted in the uterus immediately, but only later in the fall. During this interval, the female has fed, building up fat stores for hibernation. After two months the tiny cubs are born, feeding on their mother's milk until spring or early summer. The youngsters are very vulnerable, and they may remain with their mothers for about two years. Male bears may kill the young, and therefore females may mate with more than one male, resulting in litters, where the cubs may have different fathers. Because fathers want to ensure the
transmission of their own genes, they are less likely to kill cubs born of a mate. Multiple matings by females is an adaptation that reduces incidences of infanticide.

## Population Study of Brown Bears

(Wiegand et al 1998) presented a population dynamics study of the brown bear (Ursus arctos) in the Cordillera Cantabrica region of northern Spain. This brown bear population has dropped dramatically over the centuries, and environmentalists are trying to prevent their further decline. Using much data from the area and from similar bear populations elsewhere, the scientists employed two modeling techniques, an analytical technique similar to age-structured models and an individual-based (agent-based) simulation model. The analytical technique was used to obtain estimates of unknown parameters, including growth rate; to perform a sensitivity analysis for recommending the best conservation strategies; and to assess time to extinction. Individual-based simulations used the analytical estimates and data for parameters. Fitting of simulation results to actual data from 1975 to 1995 was used to refine parameter values. Fairly close agreement between the age-structured and individual-based model results increased the scientists' confidence in their results. Simulations of the population going forward reveal a great variety in possible outcomes and help to determine minimum viable population sizes. Although not as comprehensive as in Wiegand et al, projects in this module involve age-structured modeling or agent-based or cellular automaton simulations and employ much of their assumptions, data, parameters, and rules.

## Rules

The individually-based simulations of Wiegand et al 1998 incorporated a variation of the following rules:

1. Family Structure: The mother and cubs stay together with probability $\boldsymbol{i}_{i}$ until the litter is of age $i$ or until all cubs die.
2. Reproduction: A female can bear young if all the following conditions occur: She does not have a litter with her; enough territory exists; and the population has at least one adult male. With probability $f_{i}$, a female has her first litter at age $i$. A female has subsequent litters $j$ years after a family breakup (death or departure of litter) with probability $\boldsymbol{h}_{\boldsymbol{j}}$. She has a litter of size $j$ (1-4) with probability $l_{j}$, and a cub is equally likely to be a female or male.
3. Survival: The probability of death at age $i$ is $\boldsymbol{m}_{\boldsymbol{i}}^{f}$ for females and $\boldsymbol{m}_{\boldsymbol{i}}^{\boldsymbol{m}}$ for males.
4. Density dependence: A maximum of $\boldsymbol{T}_{\text {max }}$ number of females can breed in a year.
5. Environmental fluctuations: Precipitation from May to December has an impact on the environment and, thus, on the probability of a first litter, size of the litter, and mortality. In bad years, the mortality rates are

$$
\boldsymbol{m}_{\boldsymbol{i}}^{-}=m_{i}\left(1+v_{i}\right)
$$

while in good years, the mortality rates are

$$
\boldsymbol{m}_{\boldsymbol{i}}^{+}=m_{i}\left(1-v_{i}\right)
$$

where $v_{i}$ is the environmental variation at age $i$. Without environmental variation, the probability of a litter of size $j$ is $\boldsymbol{l}_{\boldsymbol{j}}$, while these probabilities are fixed at $\boldsymbol{l}_{j}^{+}$and $\boldsymbol{l}_{\boldsymbol{j}}^{-}$for good and bad years, respectively. For the projects in this module, we ignore the impact of environmental conditions on the probability of a first litter. Moreover, we assume the chance of a good or bad environmental year occurs with equal probability.

## Agent-Based Simulation

Pseudocode for the agent-based simulation follows: initialize the population's basic parameter set considering the environment, update the parameter set, based on Rule 5
for each time step, do the following:
for every female not accompanied by a litter
decide on reproduction, based on Rules 2 and 4
for every bear
decide on mortality, based on Rule 3
for every cub
decide on independence, based on Rule 1
considering the environment, update the parameter set, based on Rule 5

## Parameters

(Wiegand et al 1998) considered a variety of situations, using different parameter sets. For example, Table 1 lists mortalities, or probabilities of death, for several age groupings and parameter sets-P0, P1, P15, P2, P4, and P5.

|  |  | Parameter Set |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Symbol | Meaning | P0 | P1 | P15 | P2 | P4 | P5 |
| $m_{0}$ | mortality of cubs | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| $m_{1-4}$ | female mortality, age 1-4 | 0.15 | 0.16 | 0.165 | 0.17 | 0.20 | 0.22 |
| $m_{5-16}$ | female mortality, age 5-16 | 0.055 | 0.070 | 0.080 | 0.090 | 0.145 | 0.170 |
| $m_{17-24}$ | female mortality, age 17-24 | 0.22 | 0.24 | 0.25 | 0.26 | 0.30 | 0.33 |

Table 1 Mortalities for various parameter sets: (P0, P1, P15, P2, P4, and P5. Subadult ages are years 1-4. Adults are ages 5-25 years.

Some models do not consider constant mortalities but adjust the values using environmental variations for several scenarios, based on rainfall, as in Table 2. In such models, for a good year, a mortality, $m$, is adjusted to be $m^{+}=m(1-v)$ with the corresponding environmental variation, $v$. For example, the probability of a cub dying in a good year is reduced from $m_{0}=0.4$ to $m_{0}{ }^{+}=m_{0}\left(1-v_{0}\right)=0.4(1-0.38)=0.248$. Should the year be environmentally bad, we adjust mortality to be $m=m(1+v)$. Thus, in such a
difficult year, cub mortality is $m_{0}{ }^{-}=m_{0}\left(1+v_{0}\right)=0.4(1+0.38)=0.552$, so that there is more than a $50 \%$ chance a cub will not survive.

|  |  | Scenario |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Symbol | Meaning | S0 | S1 | S2 | S3 |
| $v_{0}$ | environmental variation, cubs | 0.00 | 0.38 | 0.38 | 0.38 |
| $v_{1-4}$ | environmental variation, age 1-4 | 0.00 | 0.00 | 0.33 | 0.66 |
| $v_{5-25}$ | environmental variation, age 5-25 | 0.00 | 0.00 | 0.16 | 0.10 |

Table 2 Environmental variations for several scenarios ( $\mathrm{S} 0-$ no variation, $\mathrm{S} 1, \mathrm{~S} 2$, S3). $v_{0}$ corresponds to mortality $m_{0 ;} v_{1-4}$ relates to $m_{1-4}$; and we use $v_{5-25}$ with $m_{5-16}$ or $m_{17-24}$. In a good year, a mortality, $m$, is adjusted to be $m^{+}=m(1-v)$ with the corresponding environmental variation, $v$; while in a bad year, $m=m(1+v)$.

Some terms lend themselves more readily to one modeling technique than another. For example, Table 3 lists symbols and values that are primarily useful for individualbased simulations. A female bear does not become mature until age 4 or 5 years, and she has a $12 \%$ chance $\left(f_{4}\right)$ of having her first litter at age 4 (Table 2). However, if she does not have a litter then, she will do so at age 5 . Any cub has equal probability $\left(s_{f}\right)$ of being female or male, and the area under consideration can support no more than 18 females breeding in a year $\left(T_{\max }\right)$.

| Symbol | Meaning | Value |
| :--- | :--- | :--- |
| $f_{4}$ | probability of first litter at age 4 | 0.12 |
| $f_{5}$ | probability of first litter at age 5 | 1.0 |
| $h_{1}$ | probability of litter 1 year after family breakup | 0.15 |
| $h_{2}$ | probability of litter 2 years after family breakup | 0.5 |
| $h_{3}$ | probability of litter 3 years after family breakup | 0.9 |
| $h_{4}$ | probability of litter 4 years after family breakup | 1.0 |
| $i_{1}$ | probability of mother and cubs together until litter age 1 | 1.0 |
| $M$ | mean litter interval | 3.1 |
| $s_{f}$ | probability that cub is female | 0.5 |
| $T_{\max }$ | maximum number of females that can breed in a year | 18 |

Table 3 Values useful for simulations
We assume that all litters become independent at age 1 year, so that $i_{0}=0.0$ and $i_{1}=$ 1.0 (Table 3). One year after a family breaks up through all the cubs dying or leaving the mother, she has a $15 \%$ chance ( $h_{1}$ ) of having another litter (Table 3). If she does not have a litter in the first year, two years after such a breakup, she has a $50 \%$ of having a litter. Four years after breakup, she will definitely have a new litter ( $h_{4}=1.0$ ) unless she has done so in the previous three years. However, the mean litter interval $(M)$ is 3.1 years.

The research article considered two techniques related to litter size-probabilities of various litter sizes, which are particularly useful in individual-based simulations, and female fecundity rates, which are valuable in age-based models. Table 4 lists such
fecundity rates, or fertilities, and probabilities of litter sizes under various environmental conditions-average, good, and bad.

|  |  | Type Year |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Symbol | Meaning | Average | Good | Bad |
| $l_{1}$ | probability of litter size 1 (for simulations) | 0.07 | 0.00 | 0.13 |
| $l_{2}$ | probability of litter size 2 | 0.55 | 0.38 | 0.74 |
| $l_{3}$ | probability of litter size 3 | 0.32 | 0.50 | 0.13 |
| $l_{4}$ | probability of litter size 4 | 0.06 | 0.12 | 0.00 |
| $Z$ | mean litter size | 2.37 | 2.75 | 2.00 |
| $M$ | mean litter interval | 3.1 |  |  |
| $f_{4}$ | probability of first litter at age 4 | 0.12 |  |  |
| $s_{f}$ | probability that cub is female | 0.5 |  |  |
| $y_{4}$ | female fertility of age 4 females, i.e. number <br> of female cubs for a 4-year old female (for <br> age-structured models) | $f_{4} s_{f} Z$ |  |  |
| $y_{5-14}$ | female fertility of age 4-14 females | $s_{f} Z / 3$ |  |  |
| $y_{16-25}$ | female fertility of age 16-25 females | $s_{f} Z / M$ |  |  |

Table 4 Probabilities of litter sizes and fertilities (Models for $y_{4}$ and $y_{5-14}$ are simplified from those in the article.)

The projects employ various combinations of the parameters in Tables 1-4.

## Projects

1. Using parameter set P 0 (Table 1) and an age-structured model for the female bears, do the following:
a. Determine the growth rate, $\lambda$. Based on the results, make a prediction about the long-term viability of the population.
b. Perform an analysis to determine to which of the mortality of fecundity variables $\boldsymbol{\lambda}$ is most sensitive. Based on the results, make recommendations for management policies to safeguard the population.
2. Develop Project 1 for one of the following parameter sets from Table 1:
a. $\quad \mathrm{P} 1$
b. P15
c. P2
d. P 4
e. P5
3. Assume the following initial population of female bears: 28 female cubs, 9 subadult females ( $1-4 \mathrm{yr}$ old), and 16 adult females ( $5-24 \mathrm{yr}$ old). Have a reasonable distribution og the population into different ages. Use parameter set P0 (Table 1) and scenario S1 (Table 2). With an age-structured model and considering
equal probabilities for good, bad, and average environmental years, determine the population distribution after 100 years. Therefore, with each step, determine the type of year and multiply the distribution by the appropriate matrix with mortalities adjusted by environmental variation, as appropriate. For the same distribution, run the model 100 times, averaging the results.
4. Develop Project 3 for one of the following parameter sets from Table 1 with scenario from Table 2:
a. P0 and S2
b. P0 and S3
c. P1 and S1
d. $\quad \mathrm{P} 1$ and S 2
e. P1 and S3
f. P15 and S1
g. P15 and S2
h. P15 and S3
i. $\quad$ P2 and S1
j. $\quad \mathrm{P} 2$ and S 2
k. P2 and S3
5. Develop Project 3 or one of the parts in Project 4. For the parameter set, the initial distribution, and scenario S 0 (no environmental variations), run the simulation to determine the population distribution after 100 years. Compare the results and discuss the impact of using environmental variation.
6. Perform an agent-based or cellular automaton simulation of the bear population employing the rules from the section "Rules" and parameter set P0 (Table 1) with no environmental variation (scenario S0) and with a time step of one year. Assume there are 25 subadult (ages 1-4 years) and adult (ages 5-25 years) females, no more than 18 female adults with litters, and 18 independent males. Run the simulation at least 100 times. Determine the following: The average distribution after 100 years; the average mortalities per year; and if extinction usually occurs, the average time until extinction. Discuss the results.
7. Develop Project 6 for one of the following parameter sets from Table 1:
a. P1
b. P15
c. P2
d. P 4
e. P5
8. Develop Project 6 taking into consideration environmental variation with scenario S1 (Table 2). Assuming equal probabilities of good, bad, and average environmental years, determine the population distribution after 100 years. With each time step (year), determine the type of year and adjust the mortalities using
environmental variations. For the same distribution, run the model 100 times, averaging the results.
9. Develop Project 8 for one of the following parameter sets from Table 1 with scenario from Table 2:
a. $\quad \mathrm{P} 0$ and S 2
b. $\quad \mathrm{P} 0$ and S 3
c. P1 and S1
d. P1 and S2
e. $\quad P 1$ and $S 3$
f. P15 and S1
g. P15 and S2
h. P15 and S3
i. $\quad \mathrm{P} 2$ and S 1
j. $\quad \mathrm{P} 2$ and S 2
k. $\quad \mathrm{P} 2$ and S 3
10. Using an agent-based or cellular automaton simulation of the bear population, determine the minimum viable population, or the smallest population that survives for at least 100 years for $95 \%$ of the simulations. Consider initial numbers of independent (non-cub) females from 10 up, in increments of 10 . Run the simulation at least 100 times for each initial number of independent females. Employ the rules from the section "Rules" and parameter set P0 (Table 1) with no environmental variation (scenario S0) and with a time step of one year. Start with a ratio of 1.8:1 adult (ages 5-25 years) to subadult (ages 1-4 years) females.
11. Develop Project 10 for one of the following parameter sets from Table 1 with scenario from Table 2:
a. P0 and S2
b. P0 and S3
c. P1 and S1
d. P1 and S2
e. P1 and S3
f. P15 and S1
g. P15 and S2
h. P15 and S3
i. $\quad \mathrm{P} 2$ and S 1
j. $\quad \mathrm{P} 2$ and S 2
k. P2 and S3

## References

Wiegand, Thorsten, Javier Naves, Thomas Stephan, and Alberto Fernandez. 1998.
"Assessing the Risk of Extinction for the Brown Bear (Ursus Arctos) in the Cordillera Cantabrica, Spain." Ecological Applications, 68(4), 1998, pp. 539-570.

